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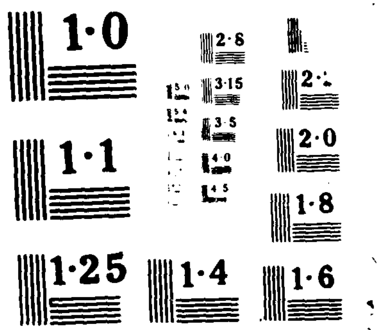
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It is shown that the basis in a class of linear programs arising from material requirements planning can be triangularized. This allows for efficient adaptation of the Simplex Method similar to those for network problems. It also suggests that for finite-loading (i.e. capacitated) MRP, a decomposition approach exploiting both subproblem structure and parallel processing can be effective for handling complex problems in multiproduct, multistage, multiperiod production systems.

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Triangularity of the Basis in Linear Programs for Material Requirements Planning

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Abstract

It is shown that the basis in a class of linear programs arising from material requirements planning can be triangularized. This allows for efficient adaptation of the Simplex Method similar to those for network problems. It also suggests that for finite-loading (i.e. capacitated) MRP, a decomposition approach exploiting both subproblem structure and parallel processing can be effective for handling complex problems in multiproduct, multistage, multiperiod production systems.

Keywords: Production and Operations Management, Material Requirements Planning, Linear Programming, Parallel Processing, Decomposition.

1. The Single-Product Infinite-Loading MRP Model

The manufacturing of a product usually consists of its assembly from parts which are themselves the products of other parts. A schematic representation of a product structure (also known as a Bill of Materials) is examplified in Figure 1. Each oblong represents an item indexed by the number in bold type. A basic assumption of the model studied in this paper is that each item except the first contributes directly to the production of only one other item, known as its parent. We shall call this the tree property of the product structure. Item 1 is the finished product. The number of units of an item required per unit production of its parent is given in parenthesis in the figure.

In Material Requirements Planning (MRP), the net demands for each item in every time period over a finite planning horizon are given. The purpose is to determine the levels of production and inventory for the items so as to meet the demands at minimum costs. For the present purpose, we assume the cost function to be linear in the production and inventory variables. Since the assembly of an item takes time, a lead time in units of time periods is specified for each item. In our example, the lead times in the following Table apply.

Item	Production Lead Time (in number of time periods)
1	1
2	1
3	2
4	1
5	1

Table 1. Production Lead Times in the Example.

We first consider the case with a single finished product (Item 1) and no production capacity or inventory storage limits on any item. The absence of capacity constraints is commonly known as infinite-loading in the MRP literature.

We call this the Single-Product Infinite-Loading MRP model (SPILMRP). The more important case of multiproduct, capacitated (finite-loading) MRP will be discussed later.

2. The Linear Programming Formulation

To formulate the above MRP problem as an LP, the following terminology is used.

Given the parameters:

N = number of items in the product structure;

T = number of time periods in the planning horizon;

d_{it} = net supply of item i at the beginning of period t;

h_{it} = unit holding cost for item i inventory in period t;

c_{it} = unit production cost for item i in period t;

$j(i)$= index of parent of item i ($i \neq 1$);

m_i = number of units of item i required per unit of its parent item $j(i)$;

L_i = production lead time for item i;

define the variables:

P_{it} = number of units of item i to be completed at the beginning of period t;

I_{it} = number of units of item i in inventory at the end of period t;

where $i = 1,\dots,N$ and $t = 1,\dots,T$ throughout.

Because of production lead times, certain variables defined above may be set to zero and eliminated from the model. For example,

$$P_{it} = 0;\ \ i=1,\dots,N,\ t=1,\dots,L_i.$$

Also, let $R_1 = T$ and $R_i = R_{j(i)} - L_{j(i)}$ for $i=2,\dots,N$ be the production horizon for item i. Then production of item i in periods $t>R_i$ will be too late to be useful.

Therefore,

$$P_{it} = 0; \; i=2,...,N, \; t=R_i+1,...,T.$$

Finally, $I_{iRi} = 0$ since allowing ending inventory will incur unnecessary holding cost.

Then the LP for the SPILMRP model can be written as (LP1):

$$\text{minimize} \sum_{i=1}^{N} \sum_{t=1}^{R_i} \{ h_{it}I_{it} + c_{it}P_{it} \}$$

$$\text{subject to} \quad -I_{i,t-1} + I_{it} - P_{it} + m_iP_{j(i),t+Lj(i)} = d_{it}; \; i=1,...,N \quad t=1,...,R_i$$

$$P_{it} = 0; \; i=1,...,N, \; t=1,...,L_i;$$

$$I_{i0} = 0; \; I_{iRi} = 0; \; i=1,...,N;$$

$$P_{it} \geq 0;$$

$$I_{it} \geq 0; \; i=1,...,N; \; t=1,...,R_i.$$

The coefficient matrix of the LP for our example is depicted in Figure 2. Denote the constraint matrix in (LP1) by M and let its dimensions be m rows by n columns.

3. Triangularity of the Basis

Observe that although (LP1) consists of essentially flow-balance type constraints, it is not a network LP. While the coefficients m_i can be considered as gain factors in a generalized network, the proportionality requirement on the production of items supplying a common parent still needs to be expressed separately. For formulations of this type of problems as networks with side constraints, see e.g Chen and Engquist [2] ,Steinberg and Napier [7], and Zahorik

et al [8]. However, the following result shows that (LP1) has a very important network-like property, namely, that any basis can be triangularized. Figure 3 shows a basis consisting of the shaded columns.

<u>Theorem</u>. A basis in (LP1) can be transformed to a lower triangular matrix by row and column permutations.

Proof. Given any m by m nonsingular submatrix B of the coefficient matrix M in (LP1), there exists either a row with a single nonzero coefficient (row singleton), or a column with a single nonzero coefficient (column singleton). Otherwise, each column has at least two nonzero coefficients. For columns corresponding to inventory variables (I-type), there must be exactly two nonzeros, a plus one and a minus one. For columns corresponding to production variables (P-type), there is a minus one and one or more m_i's. Starting from the last item, assign a multiplier to the set of rows corresponding to the same item recursively as follows. For a row containing a minus one in a P-type column with m_i's, the value is the sum over the rows containing the m_i's of m_i times the multiplier of its row. For all other rows, the value is one. Then assign the maximum value found for the set to be the multiplier for the set. Since there is no row singleton, multiplying each row by its multiplier and summing over all rows results in the zero vector. This contradicts the nonsingularity of B.

In the case of a row singleton, permute the nonzero coefficient to the upper diagonal. In the case of a column singleton, permute the nonzero to the lower diagonal. Deleting the row and column corresponding to the singleton, the remaining matrix has exactly the same structure as before and must also be nonsingular. Therefore, the same procedure can be repeated until B is lower triangularized. ◊

The assignment of multipliers is illustrated in Figure 4. Rows for the last two items do not contain a minus one in a P-type column with m_i's. The multipliers for these two sets are one's. The last three rows for Item 3 have values of 4. The first two have values of 1. Therefore the multiplier for the set is 4. Rows for Item 2 do not involve P-type columns with m_i's and their multipliers are one's. Finally, the value for each row corresponding to Item 1 is (1 + 2 x 4). Hence the multiplier for the set is 9. Total cancellation results when summing the given multiples of the rows because of sign patterns guaranteed by the absence of singletons.

Note that except for illustration, there is no need to physically permute the basis in triangularization. It suffices to identify a pivot sequence specifying which row and column to use at each step of eliminating a variable from the system of equations. The pivot sequence for the triangularization of the basis in Figure 3 is displayed in the left-most column in Figure 5. The pivots are enclosed in circles. The first pivot uses row one and column one, the second row two and column six, the third row seven and column eleven, and so on.

With a triangular basis, the major operations in the Simplex Method are greatly simplified. Both the computation of the simplex prices and the updating of a column reduce to back-substitutions. It should be remarked that basis triangularity in this case does not imply integer solutions as the m_i's may appear on the diagonal.

5. The Multi-Product Finite-Loading MRP Model

Most real production systems involve a multitude of finished products. The assembly of these products and their parts requires production capacity at every stage. When production capacities are limited, we have the finite-loading model. For a survey, see Billington et al [1]. Suppose there are K types of capacities with s_{kt} units of type k available in time period t. Let a_{ik} be the unit requirement of type k capacity in the production of item i. Then the LP for the Multi-Product Finite-Loading MRP model (MPFLMRP) is (LP2) below.

$$\text{minimize} \sum_{i=1}^{N} \sum_{t=1}^{R_i} \{ h_{it}I_{it} + c_{it}P_{it} \}$$

subject to

$$\sum_{i=1}^{N} a_{ik}P_{it} \leq s_{kt} \; ; \quad k+1,\ldots,K; \; t=1,\ldots,T \qquad \text{(LP2.1)}$$

$$-I_{i,t-1} + I_{it} - P_{it} + m_i P_{j(i),t+Lj(i)} = d_{it} \; ; \; i=1,\ldots,N \quad t=1,\ldots,R_i \qquad \text{(LP2.2)}$$

$P_{it} = 0;\; i=1,\ldots,N,\; t=1,\ldots,L_i \text{ or } t > R_i \;;$

$I_{i0} = 0;\; I_{iRi} = 0;\; i=1,\ldots,N;$

$P_{it} \geq 0;$

$I_{it} \geq 0;\;\; i=1,\ldots,N;\; t=1,\ldots,R_i.$

Here, the N items can be partitioned into mutually exclusive subsets, each corresponding to a distinct finished product. Therefore (LP2) has the block-angular structure with (LP2.1) as the coupling constraints and (LP2.2) decomposing into as many independent blocks as there are finished products. In [6] McKenney proposed to solve (LP2) using Dantzig-Wolfe decomposition [3]. Generalizing the concepts of networks, trees and paths, he devised a network-simplex type procedure to take

advantage of the triangular basis property of the subproblems.

Ongoing work in LP decomposition with parallel computers (Ho [4], Ho et al [5]) will be specialized to solve (LP2). For a 10-period MRP system with 100 products, each with 100 parts, (LP2.2) alone will have on the order of 100,000 constraints and 200,000 variables. For this reason, most previous attempts to MPFLMRP are deemed impracticable due to "prohibitive" computational requirements (see, e.g. [7]). However, multicomputers having 2^n parallel processors are becoming increasingly cost-effective. Currently, practical values of n are already between 6 and 8 (i.e. 64 to 256 processors). The power of individual processors is also well suited to handle the subproblem for one product (say, with about 1,000 constraints) if one exploits the special property discussed in this paper. Therefore, the implementation of Multi-Product, Finite-Loading Material Requirements Planning systems on parallel computers should be an important advance in production and operations management in the near future.

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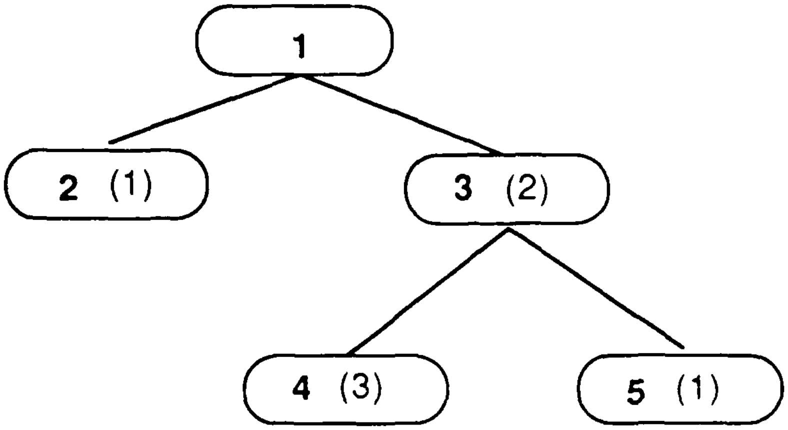

Figure 1. Example of a Product Structure

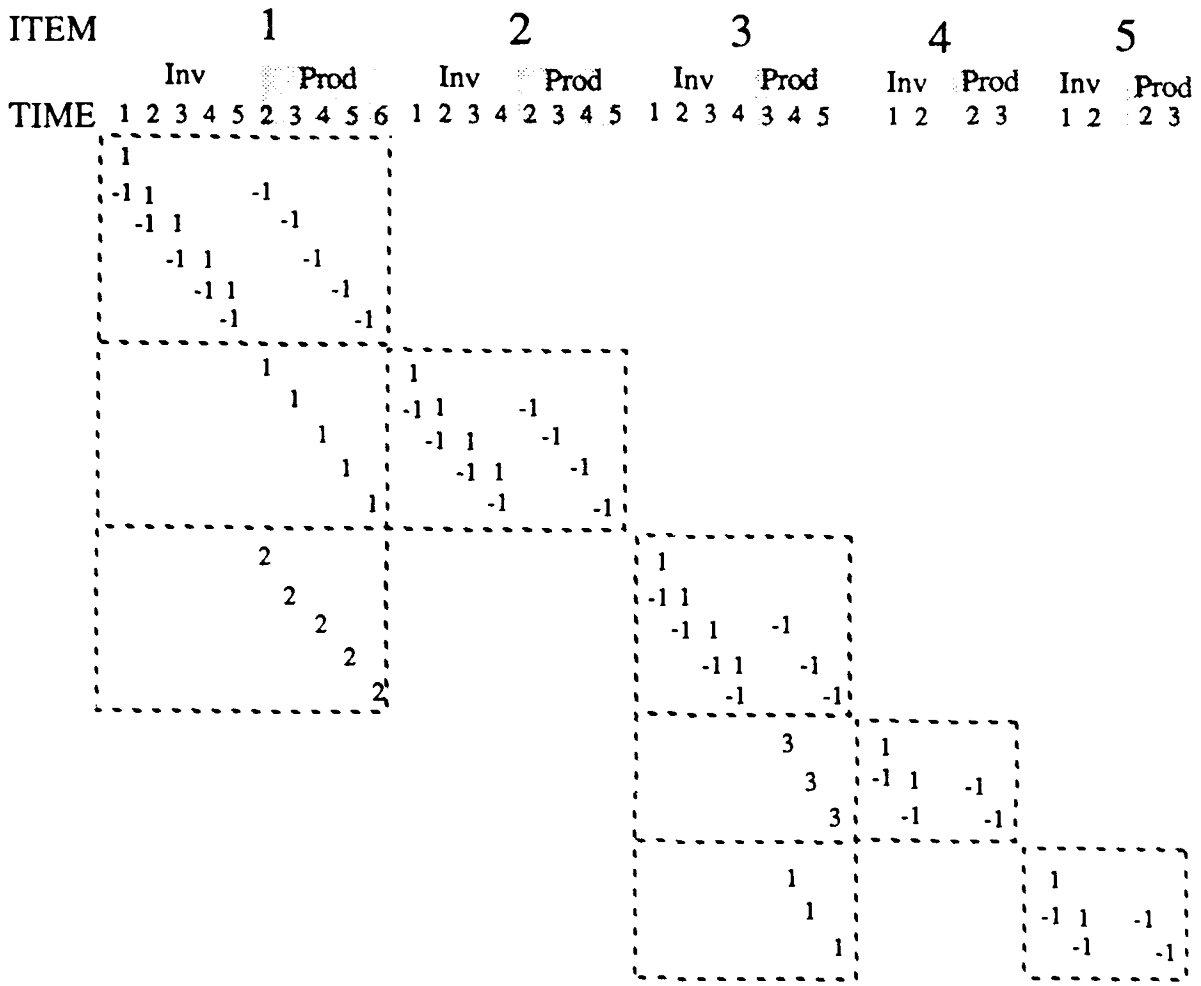

Figure 2. The LP Matrix for a Single Product MRP Problem

ITEM	1										2								3							4				5			
	Inv					Prod					Inv				Prod				Inv				Prod			Inv		Prod		Inv		Prod	
TIME	1	2	3	4	5	2	3	4	5	6	1	2	3	4	2	3	4	5	1	2	3	4	3	4	5	1	2	2	3	1	2	2	3
	1																																
	-1	1				-1																											
		-1	1				-1																										
			-1	1				-1																									
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										2												-1			-1								
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																								3		-1	1	-1					
																									3		-1		-1				
																							1							1			
																								1						-1	1	-1	
																									1						-1		-1

Figure 3. A Basis in the MRP LP

ITEM	1										2								3							4				5			
	Inv					Prod					Inv				Prod				Inv				Prod			Inv		Prod		Inv		Prod	
TIME	1	2	3	4	5	2	3	4	5	6	1	2	3	4	2	3	4	5	1	2	3	4	3	4	5	1	2	2	3	1	2	2	3
	1																																
9	-1	1				-1																											
9		-1	1				-1																										
9			-1	1				-1																									
9				-1	1				-1																								
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Row Multiplier

Figure 4. A Submatrix without Row or Column Singleton cannot be Nonsingular

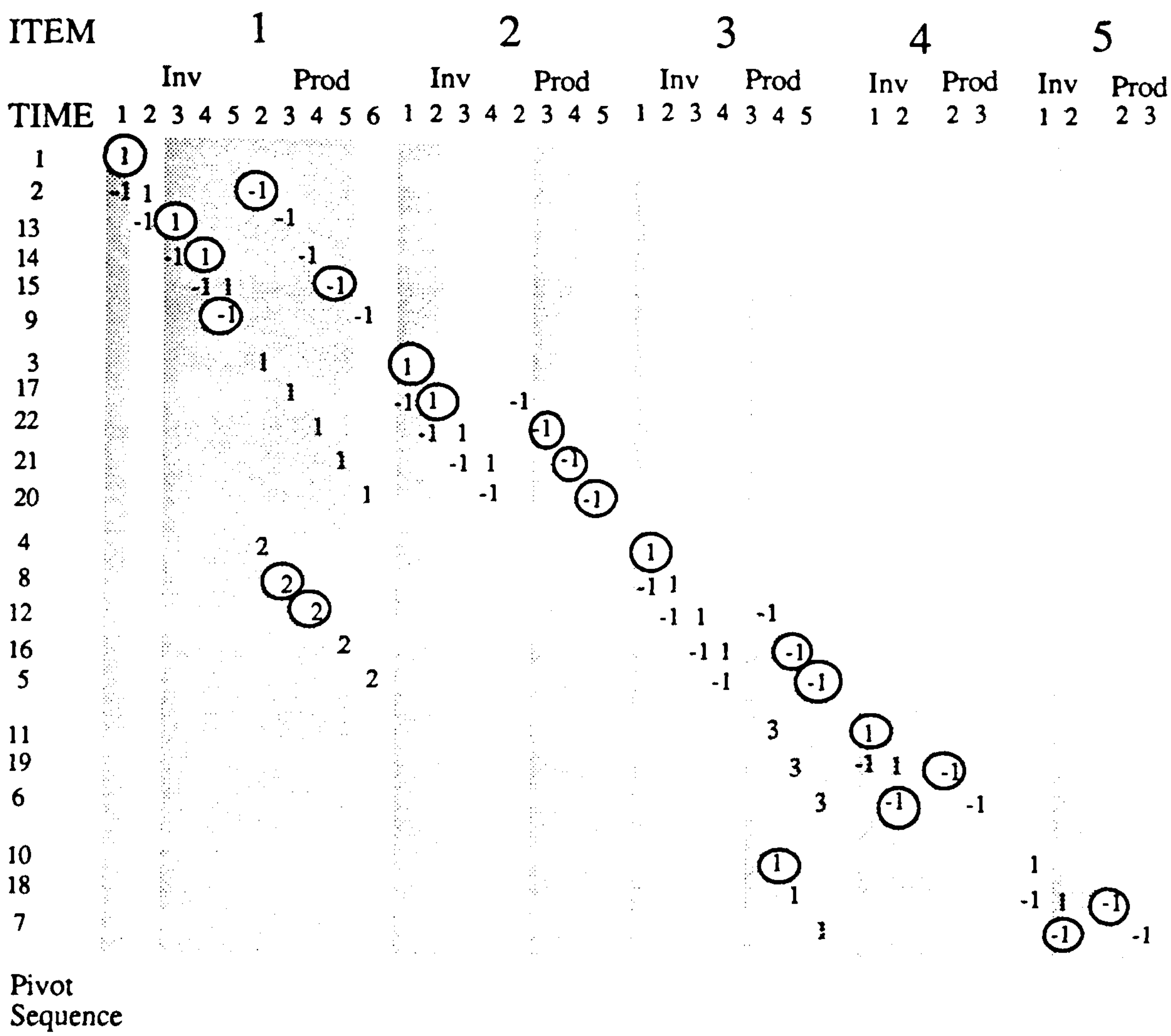

Figure 5. Pivot Sequence to Triangularize the Basis

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